Roll No. .... Total No. of Printed Pages : 4

Code No. : B02-203

Second Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper II

REAL ANALYSIS

Time : Three Hours] [Maximum Marks : 80

Note : • Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.

• Part C (Short answer type) and D (Long answer type) of each question should be answered within the word limit 200-250 and 400-450 words.

Unit-I

1. (A) Write statement of the fundamental theorem of calculus. 2

(B) Write definition of rectiable curve. 2

(C) Evaluate $\int_0^3 x \, d([x] - x)$. 4

Or

Let f be a constant function on $[a, b]$ defined by

$f(x) = k$ and α is monotonically increasing function on $[a, b]$. Then prove that $\int_a^b f \, d\alpha$ exists and $\int_a^b f \, d\alpha = k[\alpha(b) - \alpha(a)]$.

(D) (i) If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a, b]$, then prove that $f_1 + f_2 \in R(\alpha)$ and $\int_a^b (f_1 + f_2) \, d\alpha = \int_a^b f_1 \, d\alpha + \int_a^b f_2 \, d\alpha$. 6

(ii) Write definition of Integration of vector valued function. 6

Or

(i) Let $Y : [a, b] \in R^k$ be a curve. If $C \in (a, b)$, then prove that $\wedge_Y(a, b) = \wedge_Y(a, c) + \wedge_Y(c, b)$.

(ii) Write definition of unit step function.

Unit-II

2. (A) If the outer measure of a set is zero then the set is measurable. 2

(B) Define Ring of sets. 2

(C) Prove that the interval (a, ∞) is measurable. 4

Or

Show that m^* is translational inveriant.

(D) (i) Prove that the outer measure of an interval is its length. 6

(ii) Prove that the inter section and distrevence of two measurable sets are measurble. 6

Or

(i) If E_1 and E_2 are any measurable sets then prove that

$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$.

(ii) Let A be any set and $E_1, E_2, ... E_n$ a finite sequence of disjoint measureble sets. Then prove that

$$m^*\left[A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right] = \sum_{i=1}^{n} m^*(A \cap E_i).$$

Unit-III

3. (A) Write definition of semiring. 2

(B) Define Caratheodary outer measure. 2

(C) Prove that every null set is measurable. 4

Or

Let (X, B, μ) be a measure space. If $E_i \in B$, $\mu(E_1) < \infty$ and $E_i \subset E_{i+1}$, then prove that

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu(E_n).$$

(D) (i) Prove that the set function μ^* is an outer measure. 6

(ii) If $A \in a$, then prove that $\mu^*(A) = \mu(A)$. 6

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Or

(i) Let (X, S, μ) be a σ-finite measure space Σ a semiring of sets such that $S \subset \Sigma \subset B$ and $\bar{\mu}$ a measure on Σ. If $\bar{\mu} = \mu$ on S, then $\bar{\mu} = \mu$ on Σ.

(ii) A set is outer measurable iff A′ is outer measurable.

Unit-IV

4. (A) Define convex function. 2

(B) Write statement of Egoroff's theorem. 2

(C) State and prove the fundamental theorem of Integral calculus. 4

Or

State and prove that Jordan Decomposition theorem.

(D) (i) Define summable and absolutely summable. 6

(ii) A normed linear space X is complete if and only if every absolutely summable sequence is summable. 6

Or

(i) State and prove the Riesz-fischer theorem.

(ii) Write statement of Riesz theorem.

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