Roll No. .... Total No. of Sections : 3

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Annual Online Examination 2021

Code No. : A.B.C-391

B.C.A. Part III

MATHS

BCA–301

Paper I

[Calculus and Geometry]

Time : Three Hours] [Maximum Marks : 50

Note : *Section 'A' containing 10 very short answer type questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.*

Section 'A'

Answer the following Very Short Answer Type Questions in one or two sentences : **1×10=10**

1. Define upper and lower Riemann sum.
2. Write the necessory and sufficient condition for R-integrability.
3. Write the necessary and sufficient conditions for the existence of maxima or minima at a point.
4. Define local maximum.
5. Define improper integral.

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6. Write Abel's test for the convergence of integral of a product.
7. Write equation of a cone whose vertex is at origin.
8. Define right circular cylinder.
9. Find Cartesian equation of $r = a$.
10. Write the equation of the directrix of the conic

$$\frac{l}{r} = 1 + e \cos \theta.$$

Section 'B'

Answer the following Short Answer Type Questions in about 150-200 words : **3×5=15**

1. Let $f : [a, b] \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Then prove that $f \notin \mathbb{R}[0, 1]$.

Or

Prove that : $L(f) \leq U(f)$.

2. Find the critical points of $z = x^3 - y^2 - 3x$.

Or

Find the minimum distance from the origin to the plane $x + 2y - 2z = 12$.

3. Test the convergence of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

Or

Test the convergence of the integral $\int_{a}^{\infty} \frac{\sin x}{\sqrt{x}}\, dx$ where $a > 0$.

4. Find the equation of the cone whose vertex is (0, 0, 3) and base is the circle $x^2 + y^2 = 4$, $z = 0$.

Or

Find the equation of a cone whose vertex is at origin and direction cosines of its generators satisfying the relation $4l^2 + 7m^2 - 8n^2 = 0$.

5. Find the distance between the points (a, α) and $(-a, \pi + \alpha)$. What do you conclude from the results ?

Or

In a conic, prove that the semi-latus rectum is the harmonic mean between the segment of a focal chord.

Section 'C'

Answer the following Long Answer Type Questions in about 300-350 words : **5×5=25**

1. If f is define in [0, 1] by $f(x) = x$, $\forall\, x \in [0, 1]$, then prove that $f \in R\,[0, 1]$ and $\int_{0}^{1} x\, dx = \frac{1}{2}$.

Or

State and prove the fundamental theorem of Integral Calculus.

2. Find minimum value of $u = x^2 + y^2 + z^2$ having given $ax + by + c = p$.

Or

Prove that of all rectangular parallelopiped of the same volume, the cube has the least surface.

3. Examine the convergence of $\int_{0}^{1} \frac{dx}{\sqrt{x}\,(1-x)^{1/3}}$.

Or

Discuss the convergence of $\int_{0}^{1} x^{m-1}\,(1-x)^{n-1}\, dx$.

4. Find the equation of the right circular cone whose vertex is the origin, axis is z-axis and the semi-vertical angle is α.

Or

Find the equation of right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

5. Find the condition that the line $l/r = A\cos\theta + B\sin\theta$ may be a tangent to the conic $l/r = 1 + e\cos\theta$.

Or

Find the equation of the chord of the conic $l/r = 1 + e\cos\theta$, which is obtained by joining the points whose vectorial angle are 30° and 90°.

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