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Annual Examination - 2017

B.Sc.-III

MATHEMATICS

Paper-I

ANALYSIS

Max.Marks : 50

Time : 3 Hrs. **Min.Marks : 17**

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¹ãq ß Zà¾u§ý ¢§ýà¢ y¦ ¥§ý ZàÎÂà Ñv §ýLâk¥ ñ ysã ZàÎÂàà¦ §ý ¡¦§ý ytàÂà Ñ ñ

Note : Attempt one question from each unit. All questions carry equal marks.

Unit-I

ZàÎÂà-1. (¡) âÂàÈÂààvâhm ÄývÂà §ýL ÄýâÊuÊ ó½àã ZààÃm §ýLâk¥ ß

$f(x)=x^2$ kÑà¦ $-\pi \le x \le \pi$ mnà $f(x+2\pi)=f(x)$

¡m¥w ó½àã $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+----$ §ýà tâÂà ZààÃm §ýLâk¥ ñ

Find the Fourier series of the function :

$f(x)=x^2$, where $-\pi \le x \le \pi$ and $f(x+2\pi)=f(x)$. Hence find the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+-----$.

P.T.O.

(r) Ðw°²¦ qÀà¦ §ýL óâ½àuà¦ §ý ¡âsyÊ½à §ý âv¥ ¡àr¦v qÊãÕà½à §ýà §ýnÂà âvâh¥ ñ ¢y§ýL yÑàumà y¦ ây÷ §ýLâk¥ â§ý óâ½àâ

$$1-\frac{1}{4.3}+\frac{1}{4^2.5}-\frac{1}{4^3.7}+\frac{1}{4^4.9}----$$ ¡âsyàÊã Ñ¦ ñ

Write Abel's test for convergence of arbitrary series. Using this test show that the series

$$1-\frac{1}{4.3}+\frac{1}{4^2.5}-\frac{1}{4^3.7}+\frac{1}{4^4.9}----$$ is convergent.

(y) tàÂàà â§ý $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{kr } (x,y)\neq(0,0)\\ 0 & \text{kr } (x,y)=(0,0)\end{cases}$

mr qáÊsàxà y¦

$f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$ mnà $f_{xy}(0,0)$ §ýà tàÂà Öààm §ýLâk¥ ñ

Suppose that $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{when } (x,y)\neq(0,0)\\ 0 & \text{when } (x,y)=(0,0)\end{cases}$

then from definition, evaluate

$f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$ and $f_{xy}(0,0)$

Define continuity and uniform continuity of a function on a metric space (X,d). Show by an example, that every continuous function is not uniformly continuous.

(r) tàÂàvà¦ (X,d) ÀÊã§ý ytâ¦ Ñ¦ mnà ÀÊã§ý $d_1 : X \times X \to R$

âÂàÈÂààÂàyàÊ qáÊsààxm Ñ¦ ß $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$, $\forall\, x,y\in X$

mr âÀhà¢¥ â§ý ÀÊã§ý d_1 ¡àÊ d mÌu-ÀÊã§ý Ñ¦ ñ

Let (X,d) be a metric space and metric $d_1 : X \times X \to R$ is defined as follows : $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$, $\forall\, x,y\in X$. Then show that metrices d_1 and d are the equivalent metrices.

(y) Zà¾u§ý yÑm ÀÊã§ý ytâ¦ q½à Ñàmã Ñ¦, ây÷ §ýLâk¥ ñ

"Every compact metric space is complete". Prove.

---x---

(y) tàÂàvà d wàÐmâw§ý y©uà ytà°ju R qÊ yàtàÂu ÀÊã§ý Ñ mnà $A=[2,3)$ ¡àÊ $B=(3,5]$ mr âÂàÈÂààâ§ým §ýà tàÂà Öààm §ýLâk¥ ß

i) $\delta(A)$ ii) $\delta(B)$ iii) $d\left(\frac{5}{2},A\right)$

iv) $D(A,B)$ v) $d(6,B)$

kÑà δ ÍuàY, D ytà°jua §ý tÁu ÀÊã Ñ ñ

Let d be a usual metric on a set of real numbers R, and $A=[2,3)$ and $B=(3,5]$ then evaluate the following :

i) $\delta(A)$ ii) $\delta(B)$ iii) $d\left(\frac{5}{2},A\right)$

iv) $D(A,B)$ v) $d(6,B)$

where δ is diameter and D is distance between sets.

Unit-V

ZàÎÂà-5. (¡) yàm¾u ¡àÊ ¥§ý ytàÂà yàm¾u ÄývÂà §ýL qáÊsàxà, ÀÊã§ý ytâ˜ (X,d) §ý âv¥ Àããk¥ ñ ¥§ý £ÀàÑÊ½à y âÀhà¢¥ â§ý Zà¾u§ý ymm ÄývÂà; ¥§ý ytàÂà ymm ÂàÑà Ñàmà ñ

Unit-II

ZàÎÂà-2. (¡) ÄývÂà $f(x)=x^2, \forall\ x\in[0,a]$, $a>0$ §ý âv¥ ÀÎàà¢u â§ý $f\in R[0,a]$ mnà $\int_0^a x^2\,dx = a^3/3$.

For the function $f(x)=x^2, \forall\ x\in[0,a]$, $a>0$ show that $f\in R[0,a]$ and $\int_0^a x^2\,dx = a^3/3$.

(r) â»ÊÛýv qÊãÕà½à §ýà §ýnÂà âvâh¥ ñ ytà§ývÂà $\int_a^\infty \frac{\sin x}{\sqrt{x}}\,dx$ kÑà $a>0$ §ý ¡âsyÊ½à §ýà qÊãÕà½à §ýLâk¥ ñ

State the Dirichlet's test. Test the convergence of the integral $\int_a^\infty \frac{\sin x}{\sqrt{x}}\,dx$, where $a>0$.

(y) ÄývÂàã ytà§ývÂà §ý £quàªà y ây÷ §ýLâk¥ â§ý ß

Using Frullani's integral prove that :

$$\int_0^\infty \frac{\tan^{-1}ax-\tan^{-1}bx}{x}\,dx=\frac{\pi}{2}\log\frac{a}{b}$$

P.T.O.

Unit-III

ZàÎÀà-3. (¡) yâtó ÄývÂà $f(z)=u(x,y)+iv(x,y)$ §ý âwÎvâx§ý ÑàÂà §ý âv¥ ¡àwÎu§ý "§ýàÎàã-ÊãtàÂà" ¡ààÎà§ý ¡w§ývÂà ytà§ýÊ½à §ýà âvâh¥ ñ ¢y¦ ây÷ §ýLâk¥ ñ

State the necessary condition of "Cauchy-Riemann" partial differential equation for a function $f(z)=u(x,y)+iv(x,y)$ to be analytic. Prove this condition.

(r) ¥§ý âõÊhãu ÛýqàÂmÊ½à Öààm §ýLâk¥ ârÀ¡à $0,1,\infty$ §ýà §ýtÎàß $1,\ i,\ -1$ qÊ Zàâmâ j âØàm §ýÊmà Ñ ñ

Find the bilinear transformation which maps the points $0,1,\infty$ to the point $1,\ i,\ -1$ respectively.

(y) âõÊhãu ÛýqàÂmÊ½à $w=\dfrac{3z-4}{z-1}$ §ý ÐnÊ ârÀ ¡àÊ yªàm Zàyàtàu Ûýq Öààm §ýLâk¥ ñ

Find the fixed point and corresponding normal form to the bilinear transformation $w=\dfrac{3z-4}{z-1}$.

Unit-IV

ZàÎÀà-4. (¡) ÀÊã§ý ytã¦ §ýL qáÊsàxà âvâh¥ ñ ¥§ý ÀÊã§ý ytã¦ t¦ ây÷ §ýLâk¥ â§ý $\left|d(x,z)-d(y,z)\right|\le d(x,y),\forall\, x,y\in(X,d)$.

Define metric space. In a metric space (X,d)

Prove that : $\left|d(x,z)-d(y,z)\right|\le d(x,y),\forall\, x,y\in(X,d)$

(r) ytã¦ l^{∞} ysâ qáÊr÷ wàÐmâw§ý ¡Ââ§ýtà §ýà yt¦°ju Ñ ñ tàÂàvà $x=\{x_n\}_{n=1}^{\infty}$, $y=\{y_n\}_{n=1}^{\infty}$ ¢y§ý Àà Ðw¦°² ârÀ Ñ âkyt¦ ÀÊã§ý âÂàÊÂààÂàyàÊ qáÊsààxm Ñ-

$$d(x,y)=\sup\{|x_n-y_n|:n\in N\},\forall\, x,y\in l^{\infty}$$

mà âÀhà¢¥ â§ý (l^{∞},d) ¥§ý ÀÊã§ý ytã¦ Ñ ñ

Space l^{∞} is a set of all bounded real number's sequences. Suppose $x=\{x_n\}_{n=1}^{\infty}$, $y=\{y_n\}_{n=1}^{\infty}$ are two arbitary sequences points, in which the metric is defined as follows:

$d(x,y)=\sup\{|x_n-y_n|:n\in N\},\forall\, x,y\in l^{\infty}$. Then show that (l^{∞},d) is a metric space.

P.T.O.