Q.4 Define context free grammer and show that the grammer described by following grammer rules is a context free grammer. $s \rightarrow s+s|s-s|s*s||s/s|(s)|a$

OR

Show that : $L = \{a^n b^n : n \geq 1\}$ is not regular.

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Roll No.....................

Total No. of Sections : 03
Total No. of Printed Pages : 04

Code No. : 01/503(A)

First Semester Examination, Dec. 2017

M.Sc. MATHEMATICS

Paper - V

ADVANCED DISCRETE MATHEMATICS - I

Time : 3 Hrs. **Max.Marks : 80**

Note : Section 'A' consists of 10 very short answer type questions, all of which are compulsory and should be attempted first. Section 'B' consists of four short answer type questions with internal options. Section 'C' consists of four long answer type questions with internal choice.

Section - 'A'

Answer the following very short-answer-type questions in one or two sentences : (2x10=20)

Q.1 Assign a truth value to $5 < 5 \vee 5 < 6$.

Q.2 Write the equivalent proposition of $\neg(p \vee q)$

Q.3 Define lattice as partially ordered set.

Q.4 Define context sensitive grammer.

Q.5 Give the statement of Kleen's theorem.

Q.6 Show that the algebraic system $<N, *>$ where $*$ is defined by $a*b=13,\ \forall\ a,b \in N$ is a commutative semigroup.

Q.7 Draw Karnaugh map for two variables.

Q.8 Define language with an example.

Q.9 Given the truth values of P and Q as T and those of R and S as F, find the truth value of the following

$$(P \wedge (Q \wedge R)) \vee \neg(P \vee Q) \wedge (R \vee S))$$

P.T.O.

Q.10 Define regular grammer.

Section - 'B'

Answer the following questions : (5x4=20)

Q.1 Show that : $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$

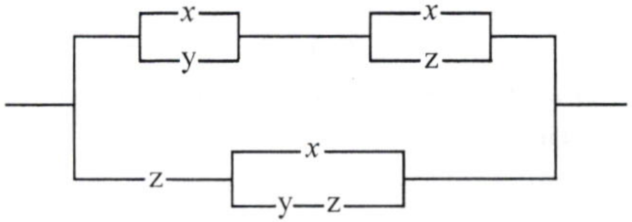
OR

For any commutative monoid $< M, * >$, show that the set of idempotent elements of M forms a submonoid.

Q.2 Show that every chain is lattice.

OR

Replace the following switching circuit by a simpler one



Q.3 State and prove the Bool's theorem.

OR

Simplify the Boolean expression $E(x_1, x_2) = x_1\ x_2 + x_1\ x_2^{1}$.

Q.4 Explain regular expressions and regular sets.

OR

State and prove pumping lemma.

Section - 'C'

Answer the following questions : (10x4=40)

Q.1 Explain the following terms and also give examples to explain them :

(i) Quantifier (ii) Universal quantifier

(iii) Existential quantifier (iv) Negative of a quantifier

OR

Define monoides with example. Let s be a non empty set and p(s) be its power set. Then prove that the algebraic struture (p(s), U) is a monoide.

Q.2 A lattice L is distributive if and only if

$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a), \forall, a, b, c \in \mathrm{L}$

OR

If a and b are arbitrary elements of a Boolean algebra B then prove that

(i) $(a+b)' = a'b'$ (ii) $(a\ b)' = a' + b'$

Q.3 Define conjunctive normal form. Write the function $(x+y+z)(xy+x'z)'$ in conjunctive normal form in which maximum number of variables are used.

OR

Use a Karnaugh map to find a minimal form of the function.

$f(x, y, z, w) = xyzw + xyzw' + xy'zw' + x'y'zw + x'y'zw'$