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Total No. of Questions : 05

Total No. of Printed Pages : 03

Code No. : B-420(B)

Annual Examination - 2017

Class BCA-III

BCA-301 Paper- I

CALCULUS & GEOMETRY

Max.Marks : 50

Time : 3 Hrs.

Min.Marks : 20

Note : Attempt any two parts from each unit. All questions carry equal marks.

Unit-I

Q-1.(a) Let $f : [a, b] \rightarrow R$ be a bounded function on $[a, b]$. Prove that $f \in R[a, b]$ iff for every $\epsilon > 0$, there exists a partition P of $[a, b]$ s.t.

$$U(p, f) - L(p, f) < \epsilon$$

(b) Let $f(x) = x^2$ on $[0, a]$, $a > 0$ show that $f \in R[0, a]$ and

$$\int_0^a x^2 dx = a^3 / 3$$

(c) State and prove the fundamental theorem of Integral Calculus.

Unit-II

Q-2.(a) Discuss the maximum or minimum values of the function :

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

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- (b) Find the minimum distance from the origin to the plane $x+2y-2z-12=0$
- (c) Find the maximum and minimum value of $u = a^2x^2 + b^2y^2 + c^2z^2$ where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$

Unit-III

- Q-3.(a) Test the convergence of $\int_2^{\infty} \frac{dx}{\sqrt{x^2-1}}$
- (b) Test the convergence $\int_a^{\infty} e^{-x} \frac{\sin x}{x^2} dx$ where $a > 0$
- (c) Prove that the integral $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$ diverges.

Unit-IV

- Q-4.(a) Show that the plane : $ax+by+cz=0$ cuts the cone $yz + zx + xy = 0$ in two perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$
- (b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the co-ordinates axes in A, B, C
Prove that the equation of the cone generated by the lines drawn from 0 to meet the circle ABC is
- $$yz \left(\frac{b}{c} + \frac{c}{b} \right) + zx \left(\frac{c}{a} + \frac{a}{c} \right) + xy \left(\frac{a}{b} + \frac{b}{a} \right) = 0$$

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- (c) Find the equation of right circular cylinder whose radius is 3 and axis is $\frac{x-1}{2} = \frac{y-3}{2} = 5-z$

Unit-V

- Q-5.(a) Prove that the polar equation of a conic is $\frac{l}{r} = 1 + e \cos \theta$ where the focus is a pole.
- (b) If Psp' is a focal chord of a conic whose focus is S and the equation is $\frac{l}{r} = 1 + e \cos \theta$ then prove that $\frac{1}{sp} + \frac{1}{sp'} = a$ constant.
- (c) Find the polar equation of a straight line which is at a distance P from the pole and the perpendicular from the pole to the line makes an angle α with the initial line.