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I Semester Examination, 2019-20

M.Sc.

MATHEMATICS

Paper IV

[Complex Analysis]

Time : Three Hours] [Maximum Marks : 80

Note : *Part A and B of each question in each unit consists of Very Short Answer Type Questions which are to be answered in one or two sentences. Part C (Short Answer Type) of each question will be answered in 200-250 words. Part D (Long Answer Type) of each question should be answered within word limit 400-450.*

Unit – I

1. (A) Write the statement of Cauchy's Goursat theorem. **2**

(B) Write the value $\int |dz|$ where L is any rectifiable are joining the points $z = \alpha$ and $z = \beta$. **2**

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(C) If c is a closed contour containing the origin inside it, prove that :

$$\frac{a^n}{|\underline{n}} = \frac{1}{2\pi i}\int_c \frac{e^{az}}{z^{n+1}}\, dz. \qquad 4$$

Or

If $f(z)$ is analytic for all finite values of z, and is bounded for all values of z, then show that it is a constant function.

(D) Let $f(z)$ be analytic in the region $|z| < \rho$ and $z = re^{i\theta}$ be any point of this region. Then show that

$$f(re^{i\theta}) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(R^2 - r^2)\, f(Re^{i\phi})}{R^2 - 2Rr\cos(\theta - \phi) + r^2}\, d\phi.$$

12

Or

Prove that :

$$\cosh\left(z + \frac{1}{2}\right) = a_0 + \sum_{n=1}^{\infty} a_n\left(z^n + \frac{1}{z^n}\right)$$

where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cosh(2\cos\theta)\cos n\theta\, d\theta.$

Unit – II

2. (A) Write the statement of maximum modulus theorem. 2

(B) Write the statement of Jordan's inequality. 2

Or

Let the metric ρ be defined as in $\rho(f, g) = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n \frac{\rho_n(f, g)}{1 + \rho_n(f, g)}$.

If $\varepsilon > 0$ is given, then show that there is a $\delta > 0$ and a compact set $K \subset G$ such that for $f, g \in C(G, \Omega)$

$\sup\{d(f(z), g(z) : z \in k\} < \delta \Rightarrow \rho(f, g) < \varepsilon.$

(D) To state and prove Montel's theorem. 12

Or

If $|z| \leq 1$ and $P \geq 0$, then show that

$$|1 - E_P(z)| \leq |z|^{p+1}.$$

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(C) If $f(z)$ is analytic in a domain $|z| < 1$ and satisfies the conditions $|f(z)| \leq 1, f(0) = 0$, then show that $|f(z)| \leq |z|$ and $|f'(0)| \leq 1$.

4

Or

If c is the arc $\theta_1 \leq \theta \leq \theta_2$ of the circle $|z - a| = r$ and if $\lim_{z \to a} (z - a) f(z) = A$, then show that

$$\lim_{r \to 0} \int_c f(z)\, dz = iA(\theta^2 - \theta_1).$$

(D) If $f(z)$ is analytic within and on a closed contour C except at a finite number of poles and has no zero on C, then show that

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}\, dz = N - P$$

where N is the number of zeros and P the number of poles inside C. **12**

Or

Apply the calculus of residues to prove that :

$$\int_0^{\infty} \frac{\sin \pi x}{x(1-x^2)}\, dx = \pi.$$

Unit – III

3. (A) Define fixed points of a bilinear transformation. **2**

(B) Write the statement of sufficient condition for $w = f(z)$ to represent a conformat mapping. **2**

(C) Show that cross ratios are invariant under a bilinear transformation. **4**

Or

Find the fixed points and the normal form of the bilinear transformation

$$\omega = \frac{(2+i)\,z-2}{z+i}.$$

(D) To find all the bilinear transformations which maps the half plane $I(z) \geq 0$ on to the unit circular disc $|w| \leq 1$. **12**

Or

Discuss the transformation $\omega = z^2$.

Unit – IV

4. (A) Defined locally bounded set. **2**

(B) Write the statement of Weierstrass's theorem. **2**

(C) If $F \subset C(G, \Omega)$ is equicontinuous at each point of G, then show that F is equicontinuous over each compact subset of G. **4**