subject to the constraint

$$4x^2 + y^2 = 1. \qquad 12$$

Or

Divide a number n into three parts x, y, z such that $ayz + bzx + cxy$ shall have a maximum or minimum, and determine which it is.

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Roll No. ....

Total No. of Sections : 4

Total No. of Printed Pages : 6

Code No. : 01/203

I Semester Examination, 2019-20

M.Sc.

MATHEMATICS

Paper II

[Real Analysis]

Time : Three Hours] [Maximum Marks : 80

Note : Part A and B of each question in each unit consists of Very Short Answer Type Questions which are to be answered in one or two sentences. Part C (Short Answer Type) of each question will be answered 200-250 words. Part D (Long Answer Type) of each question should be answered within the word limit 400-450.

Unit–I

1. (A) Write the definition of point wise convergence of a sequence of functions. **2**

(B) Write statement of Weierstrass's M-test for uniform convergence. **2**

(C) Show that the series

$$\sin x+\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x+....$$

converges uniformly in $0 < a \leq x \leq b < 2z$. 4

Or

Prove that the limit function of uniformly convergent sequence of continuous functions is itself continuous.

(D) State and prove the Weierstrass approximation theorem. 12

Or

State and prove the Abel's test for uniform convergence.

Unit–II

2. (A) Write the most important fact about a power series. 2

(B) Find the radius of convergence of the series

$$1+x+\frac{x^2}{\underline{|2}}+\frac{x^3}{\underline{|3}}+\frac{x^4}{\underline{|4}}+.... \qquad 2$$

(D) State and prove the Inverse function theorem. 12

Or

State and prove the Taylor's theorem, and define line segment.

Unit–IV

4. (A) Write statement of partitions of unity. 2

(B) Write definition of the integral of 1-Form. 2

(C) Explain Lagrange's multiplier method. 4

Or

Find the rectangle of perimeters which has maximum area.

(D) Determine the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 + \frac{3\sqrt{3}}{2}xy$$

(C) Determine the radius of convergence of the following power series : **4**

(i) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n+1}}$

(ii) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)^B}, (B > 1)$.

Or

Show that

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots.. = \log 2.$$

(D) (i) State and prove the Abel's theorem (First form) for power series. **8**

(ii) If the two real power series $\Sigma a_n x_n$ and $\Sigma b_n x_n$ have a radius of convergence $R > 0$ and converge to the same function in $(-R, R)$, then prove that the two series are identical. **4**

Or

State and Prove the Riemann's theorem. **12**

Unit–III

3. (A) Write definition of linear transformation. **2**

(B) Write state of chain rule. **2**

(C) If $A, B \in L(R^n, R^m)$ and C is a scalar, then prove that

$$\| A + B \| \leq \| A \| + \| B \|,$$

$$\| CA \| = | C | \| A \|$$

with the distance between A and B defined as $\| A - B \|$, $L(R^n, R^m)$ is a metric space. **4**

Or

Let f maps a convex open set $E \subset R^n$ into R^m, f be differentiable in E and there be a real number M such that

$$\| f'(x) \| \leq M$$

for every $x \in E$, then prove that

$$| f(b) - f(a) | \leq M | b - a |, \text{ for all } a \in E, b \in E.$$