Roll No. .....

Total No. of Sections : 4

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Code No. : 01/103

I Semester Examination, 2019-20

M.Sc.

MATHEMATICS

Paper I

[Advanced Abstract Algebra]

Time : Three Hours] [Maximum Marks : 80

Note : *Part A and B of each question in each unit consist of very short answer type questions which are to be answered in one or two sentences. Part C (Short answer type) of each question will be answered 200-250 words. Part D (Long answer type) of each question should be answered within the word limit 400-450.*

Unit–I

1. (A) Write the difference between Subnormal series and Normal series. **2**

(B) Given an example of camposition series. **2**

(C) Prove that the subgroup of a solvable group is also solvable. **4**

(D) State and prove the Jordan-Hölder theorem. **12**

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Unit–II

2. (A) Define algebraically closed field. **2**

(B) Write the difference between separable and inseparable extensions. **2**

(C) Let $F \subseteq E \subseteq K$ be fields. If K is a finite extension of E and E is a finite extension of F, then prove that K is a finite extension of F and

$$[K : F] = [K : E][E : F].$$

(D) Show that $x^3 - 2 \in Q[x]$ is irreducible over Q. Find an extension K of Q having all roots of $x^3 - 2$ such that $[K : Q] = 6$. **12**

Unit–III

3. (A) Define Normal extension of a field. **2**

(B) Show that $Q(5\sqrt{7})$ is a normal extension over Q. **2**

(C) If $f(x), g(x) \in F[x]$, then show that

$(f(x) + g(x))' = f'(x) + g'(x)$. **4**

(D) Define splitting field of a Polynomial. Show that the splitting field of $x^4 + 1$ over Q is $Q(\sqrt{2}, i)$ whose degree over Q is 4.

Unit–IV

4. (A) Define F-automorphism. **2**

(B) Define fixed field. **2**

(C) Let E be a finite extension of a field F, then show that $G(E/F)$ is a finite group and **4**

$$| G(E/F) | \leq [E : F].$$

(D) State and prove fundamental theorem of Galois theory. **12**

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