Or

Find the equation of the cylinder whose generators are parallel to Z-axis and intersection the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$.

5. Show that the two conics

$$\frac{l_1}{r} = 1 + e_1 \cos \theta \text{ and } \frac{l_2}{r} = 1 + e_2 \cos (\theta - \alpha)$$

will touch one another if

$$l_1^2(1-e_2^2) + l_2^2(1-e_1^2) = 2l_1l_2(1-e_1e_2 \cos \alpha).$$

Or

Show that the condition that the chord cut off by the conic $\frac{1}{r} = 1 + e \cos \theta$ from the straight line $\frac{1}{r} = a \cos \theta + b \sin \theta$ subtend a right angle at the pole is $(la - e)^2 + l^2b^2 = 2$.

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Roll No. ....

Total No. of Sections : 3

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Annual Examination, 2020

B.C.A. Part III

MATHEMATICS

Paper I

[Calculus and Geometry]

Time : Three Hours] [Maximum Marks : 50

Note : *Section 'A', containing 10 very short answer type questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.*

Section 'A'

Answer the following very short answer type questions in one or two sentences. **1×10=10**

1. Write Weierstrass's (second) Mean Value Theorem.

2. Write the Fundamental Theorem of Integral Calculus.

3. Write Lagrange's condition for two independent variables.

4. Write the name of the rectangular solid of maximum volume that can be inscribed in a sphere.

5. Write the values of n for which the integral $\int_a^\infty \frac{dx}{x^n}$, $a > 0$ converges and diverges.

6. Write Dirichlet's test for the convergence of improper integral of second kind.

7. Write the general equation of the cone of the second degree which passes through the co-ordinate axes.

8. Write the condition that the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

will have three mutually perpendicular tangent planes.

9. Write the Cartesian equation of the curve $x^2 + y^2 = 2ax$.

10. Write the polar equation of a conic whose focus is pole, eccentricity is e and latus rectum is $2l$.

Section 'B'

Answer the following short answer type questions with word limit 150–200. **3×5=15**

1. If f be a real valued bounded function defined on

Or

If $f \in \mathbf{R}[a, b]$, then $|f| \in \mathbf{R}[a, b]$ and

$$\left| \int_a^b f \right| \leq \int_a^b |f|.$$

2. Find the maximum and minimum values of

$$u = a^2x^2 + b^2y^2 + c^2z^2,$$

where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$.

Or

Find the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

3. Test the convergence of $\int_0^1 \frac{dx}{x^3(1+x^2)}$.

Or

Prove that $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ converges.

4. Find the equation of the right circular cone with vertex at (1, – 2, – 1), semi-vertical angle 60° and the axis $\frac{(x-1)}{3} = -\frac{(y+2)}{4} = \frac{(z+1)}{5}$.

$[a, b]$ and m and M are the infimum (glb) and supremum (lub) of $f(x)$ in $[a, b]$, then for any partition P of $[a, b]$, we have

$$m(b-a) \leq L(P, f) < U(P, f) \leq M(b-a).$$

Or

Let $f(x) = x^2$ on $[0, a]$, $a > 0$. Show that

$$f \in R[0, a] \text{ and } \int_0^a x^2 dx = \frac{a^3}{3}.$$

2. Find all the maximum and minimum values of the function given by $f(x, y) = xy(a - x - y)$.

Or

Find the maximum or minimum value of $x^4 + y^4 + z^4$, where $xyz = c^3$.

3. Test the convergence of the integral $\int_0^\infty \frac{\cos x}{1+x^2} dx$.

Or

Test the convergence of $\int_a^\infty \frac{\sin x}{x^n}$, $n > 0$.

4. Find the equation of the cone whose vertex is (5, 4, 3) and base curve $3x^2 + 2y^2 = 6, y + z = 0$.

Or

Find the equation of the conc whose vertex is origin and which passes through the curve

$$ax^2 + by^2 + cz^2 = 1,$$

$$lx + my + nz = p.$$

5. Show the equation $\frac{l}{r} = f(\theta)$ where

$$f(\theta) = a \cos(\theta + \alpha) + b \sin(\theta + \alpha)$$

represent a straight line.

Or

Prove that the equations $\frac{l}{r} = 1 - e \cos\theta$ and $\frac{l}{r} = -1 - e \cos\theta$ represent the same conic.

Section 'C'

Answer the following long answer type questions with word limit 300–350. **5×5=25**

1. If $f, g : [a, b] \rightarrow \mathbf{R}$ and $f, g \in \mathbf{R}[a, b]$, then $f + g \in \mathbf{R}[a, b]$.