Roll No. .... Total No. of Printed Pages : 4

Code No. : B02-303

Second Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper - III

GENERAL AND ALGEBRAIC TOPOLOGY

Time : Three Hours] [Maximum Marks : 80

Note :
- Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.
- Part C (Short answer type) and D (Long answer type) of each question should be answered within 200-250 and 400-450 words.

Unit-I

1. (A) Define Tychonoff product topology. 2

(B) Explain distinguish points. 2

(C) Let $T_1 = \{\phi, \{1\}, X_1\}$ be a topology on $X_1 = \{1, 2, 3\}$ and $T_2 = \{\phi, X_2, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$. be a topology on $X_2 = \{a, b, c, d\}$. Find a base for the product topology. 4

Or

Prove that the product space of two Hausdorff spaces is Hausdorff.

(D) Let (X, T) be the product space of (X_1, T_1) and (X_2, T_2). Let $\pi_1 : X \to X_1$, $\pi_2 : X \to X_2$ be the projection maps on the first and second co-ordinate spaces respectively. Let $f : Y \to X$ be another map. Show that f is continuous iff π_1 of and π_2 of are continuous maps. 12

Or

Prove that the product of completely regular spaces is completely regular.

Unit-II

2. (A) Define Local finiteness. 2

(B) Define paracompactness. 2

(C) Let $F_i : (X_i, T) \to (Y, V)$ be a continuous onto map $\forall\, i \in I$. V being largest topology for Y. Prove that the map $g : Y \to Z$ is continuous iff the composition map is continuous. 4

Or

Let X be a regular space with a basis β that is countably locally finite. Then prove that X is normal and every closed set in X is a G_g set in X.

(D) Let $\{f_i : X \to Y_i \ \forall\ i \in I\}$ be a family of continuous functions which distinguishes points from closed sets, then prove that the corresponding evaluation map is an embedding of X into the product space $\Pi_{i \in I} Y_i$. 12

Or

State and prove Urysohn Metrization Theorem.

Unit-III

3. (A) Give an example of directed set. 2

(B) Define convergence of filters. 2

(C) Let (X, T) be a topological space and $Y \subset X$, then show that Y is T open iff no net in X – Y can converge to a point in Y. 4

Or

Prove that a net in a set X is ultranet iff the filter it generates is an ultrafilter.

(D) Let (X, T) be a topological space and $Y \subset X$. Then prove that a point $x_0 \in X$ is a limit point of Y iff these exists a net in $Y - \{x_0\}$ converges to x_0. 12

Or

Prove that a filter F on a set X is an ultrafilter iff for any two subsets A, B of X such that $A \cap B \in F$, we have either $A \in F$ or $B \in F$.

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Unit-IV

4. (A) Explain the fundamental group. 2

(B) Define convering space. 2

(C) Let $x_0, x_1 \in X$. If these is a path in X from x_0 to x_1 then prove that the groups $\pi_1 (X_1, x_0)$ and $\pi_1 (X_1, x_1)$ are isomorphic. 4

Or

Prove that $\simeq, \simeq_p$ is an equivalence relation.

(D) Prove that the operation * is well defined on path homotopy classes and it satisfies associativity, identify and inverse properties. 12

Or

State and prove that Fundamental Theorem of Algebra.

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