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Fourth Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper II

MECHANICS

Time : Three Hours] [Maximum Marks : 80

Note :
- Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.
- Part C (Short answer type) and D (Long answer type) of each question should be answered within the word limit 200-250 and 400-450.

Unit-I

1. (A) Define constrained motion with one example only. 2

(B) Write the transformation equation for a system of N particles, from cartesian coordinate to generalized coordinates. 2

(C) Derive the Hamilton's cannonical equations of motion for the given position coordinates q_j, momenta p_j and time t. 4

P. T. O.

Or

Obtain the lagrange's equation of motion of second kind for the conservative system.

(D) Derive the Routh's equation of motion from Lagrangian L $(q_1, ... q_n, \dot{q}_1, ... \dot{q}_n, t)$. 12

Or

State and prove Donkin's theorem.

Unit-II

2. (A) Define Poisson Bracket. 2

(B) Define Geodesic. 2

(C) Show that minimum surface of revolution is generated by a catenary. 4

Or

Show that the path of shortest distance between two points on a plane is straight line.

(D) State and prove principle of least action. 12

Or

Show that the poincare-cartan integral is invariant.

Unit-III

3. (A) State Whittaker's equation. 2

(B) State Lee-Hwa Chung's theorem. 2

(C) Show that the transformation :

$$Q = \sqrt{2q}\, e^{a} \cos p$$

$$P = \sqrt{2q}\, e^{-a} \sin p$$

is a canonical transformation. 4

Or

For a certain canonical transformation, it is given that,

$$Q = \sqrt{q^2 + p^2}$$

and $$F = \frac{1}{2}(q^2 + p^2)\tan^{-1}\frac{q}{p} + \frac{1}{2}qp,$$

where $$P = -\frac{\partial F}{\partial Q}.$$

Then find the value of P (q, p).

(D) Show that the Lagrange's bracket is invarient under the canonical transformation. Further show that $\{q_i, q_j\} = 0$, $\{p_i, p_j\} = 0$ and $\{q_i, p_j\} = \delta_{ij}$ for Lagrange Bracket. 12

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Or

Prove that $\sum_{l=1}^{2n} \{u_l, u_i\}.[u_l, u_j] = \delta_{ij}$ where $\{u_l, u_i\}$ is Lagrange and $[u_l, u_i]$ is Poisson Bracket. (It is the relation between Lagrange and Poisson Brackets.) Here δ_{ij} denotes the Kronecker delta.

Unit-IV

4. (A) State the 'Gauss theorem' for total normal attraction. 2

(B) State the 'Laplace theorem' for the potential of an attracting mass. 2

(C) Show that the family of right circular cones $\frac{x^2 + y^2}{z^2}$ = constant, is a possible family of equi-potential surfaces. 4

Or

Find the work done by mutual attraction of the particles of a self-attracting system, when particles are brought from an infinite distance from one-another to the position they occupy in the given system.

(D) Find the attraction of a "uniform circular disc" (plate) of radius a and small thickness k, at a point on the axis of the disc at a distance p from its centre. 12

Or

Find the attraction of a thin uniform rod AB at an external point P.

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