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Code No. : B04-103

Fourth Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper I

FUNCTIONAL ANALYSIS - II

Time : Three Hours] [Maximum Marks : 80

Note : • Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.

• Part C (Short answer type) and D (Long answer type) of each question should be answered within the word limit 200-250 and 400-450.

Unit-I

1. (A) Define closed graph. 2

(B) If X be a linear space then define entension f to X. 2

(C) Let T be a closed linear map of Banach space X into Banach space Y then T is continuous. 4

Or

Let X be a complex linear space and let M be a linear subspace of X. Let p be a seminorm on X and f be a linear functional defined on M such that $|g(x)| \leq p(x)$ for all $x \in X$.

(D) Let X be a normed linear space over field K and M be a linear subspace of X. Suppose that x_0 be vector not in M and $d(X_0, M) = d > 0$. Then their exists $g \in X^*$ such that $g(M) = \{0\}$, $g(x_0) = d$ and $\| g \| = 1$. 12

Or

State and closed graph theorem.

Unit-II

2. (A) Define uniformly convex. 2

(B) Define orthonormal set. 2

(C) Every innerproduct space is normed space. 4

Or

State and prove parallelogram law.

(D) State and prove Bessel's inequality. 12

Or

Let S = {x_1, x_2, ...} be linearly independent sequence in an inner product space. Then there exists an orthonormal sequence T = {y_1, y_2, ...} such that L(s) = L(T).

Unit-III

3. (A) Define adjoint operator. 2

(B) Give the definition of orthogonal complement. 2

(C) Let M be a linear subspace of Hilbert space H. Then M is closed if and only if $M = M^{11}$. 4

Or

Let M be a closed subspace of a Hilbert space H and $x \in$ H. Then there exist unique element $y \in$ M and $z \in M^1$ such that $x = y + z$ *i.e.*, $H = M + M^1$.

(D) A closed convex subset c of a Hilbert space H contains a unqiue vector of smallest norm. 12

Or

Every Hilbert space H is reflexive.

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Unit-IV

4. (A) Define self adjoint operator. 2

(B) Define normal and unitary operator. 2

(C) Every positive operator is self adjoint. 4

Or

If T_1 and T_2 are normal operators on a Hilbert space H with property that either commutes with adjoint of the other than $T_1 + T_2$ and T_1T_2 are normal.

(D) Let S and T be unitary operators on Hilbert space H. Then, 12

(i) T is isometric

(ii) $\| T \| = 1$ provided $H \neq \{0\}$

(iii) T is normal

(iv) T is unitary

(v) T^{-1} (= T*) is unitary

Or

If T is positive operator on a Hilbert space H then I + T is non singular.

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