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Code No. : B02-103

Second Semester Online Examination, May-June, 2022

M.Sc. MATHEMATICS

Paper I

ADVANCED ABSTRACT ALGEBRA II

Time : Three Hours] [Maximum Marks : 80

Note :
- Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.
- Part C (Short answer type) and D (Long answer type) of each question should be answered within the word limit 200-250 and 400-450 words.

Unit-I

1. (A) Define basis with an example. 2

(B) Define primary modules with an example. 2

(C) Show that a non-zero finitely generated vector space always admits a finite basis. 4

Or

Show that every submodule of a noetherian module is finitely generated.

(D) Define nil ideal, nilpotent ideal and show that any nilpotent left ideal in an artinian ring is always nilpotent,. 12

Or

State and prove Hilbert Basis theorem.

Unit-II

2. (A) Define regular transformation. 2

(B) If $T_1, T_2 \in$ Hom (U, V) then show that $(T_1 + T_2) \in$ Hom (U, V). 2

(C) Is $T(\alpha, \beta) = (\alpha + \beta, \alpha - \beta, \beta), \forall (\alpha, \beta) \in V_2(R)$ a linear transformation ? 4

Or

Find the matrix of T relative to the basis $\{\alpha_1, \alpha_2\}$ where $\alpha_1 = (1, 1)$, $\alpha_2 = (-1, 0)$ and $T(x, y) = (4x - xy, 2x + y)$.

(D) Show that rank (T) + nullity (T) = dim V, where $T : U \to V$. 12

Or

Let V be the vector space and $T \in A(V)$ is defined as $(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3)\, T = \alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2$. Compute the matrix of T is the basis A, $1 + x$, $1 + x^2$, $1 + x^3$ and find CAC^{-1}.

Unit-III

3. (A) Define companian matrix with example. 2

(B) Define trace of a matrix with example. 2

(C) Determine all invariant subspaces of $A = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$ on R^2. 4

Or

Show that similar matrices have the same eigen values.

(D) Find the canonical nilpotent forms of the following matrix $\begin{bmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ 2 & -1 & -1 \end{bmatrix}$. 12

Or

If dim V = n and if T $\in$ A(V) has all its characteristics roots in F then show that T satisfies a polynomial of degree n over F.

Unit-IV

4. (A) Define row module of a matrix with an example. 2

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(B) Find rational canonical form of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 2 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{bmatrix}$. 2

(C) Show that by an example that not every matrix is similar to a diagonal matrix. 4

Or

Show that T or M is a submodule of M.

(D) Suppose :

(i) V is a finite dimensional vector space over a field F.

(ii) T $\in$ Hom_F (V, V).

(iii) $f(x) = g(x)\, h(x)$ is F $[x]$ such that $[g(x), h(x)] = 1$.

Show that $f(T) = 0$ if and only if

$$V = \ker g(T) \oplus \ker h(T).$$

Or

Describe the process of diagonalization of A, $m \times n$ matrix over P | D R such that

A = diag $(a_1, a_2, ... a_r, 0, 0 .... 0)$ $a_i \mid a_{i+1}$.

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