Roll No. .... Total No. of Printed Pages : 6

Code No. : B02-503

Second Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper - V

ADVANCED DISCRETE MATHEMATICS - II

Time : Three Hours] [Maximum Marks : 80

Note :
- Part A and B of each equation in each unit consist of very short answer type questions which are to be answered in one or two sentences.
- Part C (Short answer type) and D (Long answer type) of each question should be answered within the word limit 200-250 and 400-450 words.

Unit-I

1. (A) Define complete graph with example. 2

(B) Define N-cube graph with example. 2

(C) Show that the total number of odd degree vertices of a (p-g) graph is always even. 4

Or

If every region of a simple planar graph with n-vertices and e-edges embeded in a plane is

bounded by k-edges, then show that $e = \frac{k(n-2)}{(k-2)}$.

(D) (i) A simple graph G is a spanning tree if and only if G is connected. 6

(ii) Show that if a tree has exactly two pendent vertices, the degree of every other vertex is two. 6

Or

(i) If G is a connected planar graph with n-vertices and r-region, then show that

$$n - e + r = 2.$$

(ii) Suppose G is graph with 1000 vertices and 3000 edges. Is G planar ?

Unit-II

2. (A) Define Regular graph with example. 2

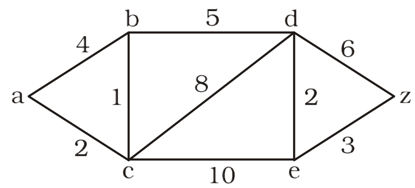
(B) How many vertices and edges in the graph $k_{m,\ n}$. 2

(C) Define tree traversal and explain kinds of tree traversal. 4

Or

Form a binary search tree for the data 16, 24, 7, 5, 8, 20, 40, 3 in the given order.

(D) Define shortest path with application. Use Dijkstra's algorithm to find shortest path from *a* to *z* of the following graph. 12



Or

(i) Show that the maximum number of lines among all *p* point graphs with no triangles is $\frac{p^2}{4}$.

(ii) The following statements are equivalent for a connected graph G (i) G is Eulerian (ii) Every point of G has even degree (iii) The set of lines of G be partitioned into cycles.

Unit-III

3. (A) Define finite state machine. 2

(B) Define *k*-equivalent states with example. 2

(C) Design a finite state machine M which can add two binary numbers. 4

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Or

Let M = (S, I, O, *f*, *g*, S_0) be a finite state machine. Then the relation "*k* equivalence on the sets of all states of M" is an equivalence relation.

(D) Find π_0, π_1 and π_2 for the following finite state machine, also define *o*-equivalent. 12

State	**Input**		**Output**
	0	1	
S_0	S_1	S_5	0
S_1	S_0	S_5	0
S_2	S_6	S_0	0
S_3	S_7	S_1	0
S_4	S_0	S_6	0
S_5	S_7	S_2	1
S_6	S_0	S_3	1
S_7	S_0	S_2	1

Or

Minimize finite state machine M of the following also define *o*-equivalent.

State	Input		Output
	0	1	
S_0	S_3	S_1	1
S_1	S_4	S_1	0
S_2	S_3	S_0	1
S_3	S_2	S_3	0
S_4	S_1	S_0	1

Unit-IV

4. (A) Define turing machine with example. 2

(B) Define finite state language. 2

(C) Show that the language $L = \{a^k b^k : k \geq 1\}$ is not a finite state language. 4

Or

Prove that for any trasition function δ and for any two input strings x and y,

$$\delta\ (a_1,\ xy) = \delta\ (\delta\ (a_1,\ x),\ y).$$

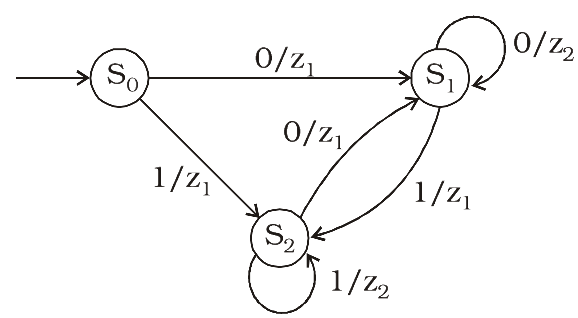
(D) Define Mealy machine with example. Construct Mealy machine which is equivalent the Moore machine given in the table : 12

Poesent State	Next State		Output
	$a = 0$	$a = 0$	
S_0	S_3	S_1	0
S_1	S_1	S_2	1
S_2	S_2	S_3	0
S_3	S_3	S_0	0

Also construct the diagram of Mealy machine.

Or

Define Moore machine with example. Mealy machine is given that, construct a Moore machine equivalent to this Mealy machine.



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