Roll No............... **Total No. of Printed Pages : 4**

Code No. : B02/403

Second Semester Online Examination, May-June, 2022

M. Sc. MATHEMATICS

Paper IV

COMPLEX ANALYSIS - II

Time : Three Hours] [Maximum Marks : 80

Note : *Part A and B of each question in each unit consist of 'very short answer type question' which are to be answered in one or two sentences. Part C 'Short answer type' and D 'Long answer type' of each question should be answered within the word limit mentioned.*

UNIT-I

1. (A) Define Euler's Gamma function. **2**

(B) Define Riemann Zeta function. **2**

(C) Find the residue at the poles of $\Gamma(Z)$.

(word limit 200-250) **4**

OR

Prove that the zeta function G(Z) can be extended to a meromorphic in the whole plane with only a simple pole at $Z \neq 1$ and $\text{Res}_{Z=1}$,G(Z) for $Z \neq 1$, Z satisfies Riemann's functional equation.

(D) To prove:

$$\sqrt{\pi}\ \Gamma(2Z) = 2^{2z-1}\sqrt{Z}\left(Z+\frac{1}{2}\right)$$

(word limit 400-450) **12**

OR

State and Prove Mittag-Leffler Theorem.

UNIT-II

2. (A) Define Analytic Continuation. **2**

(B) Write the statement of Mean-value Theorem for Harmonic function. **2**

(C) Prove that there cannot be more than one Analytic contrinuation of a function $f(z)$ into the same domain.

(word limit 200-250) **4**

OR

Let $D = \{z : |z| <| \}$ be the unit disc with the boundry $\partial D = \{z : |z| = 1\}$ and let $f : \partial D \to R$ be a continuous function. Then there is a continuous function $u : \overline{D} \to R$ such that $4(Z) = f(0)$ for $z \in \partial D$.

(D) Prove that if the radim of on convergence of the power series $f(Z) = \sum_{n=p}^{0} a_n (Z - Z_0)^n$ is non-zero finite, then $f(Z)$ has at least sigularity on the circle of convergence. *(word limit 400-450)* **12**

OR

State and prove monodromy theorem.

UNIT-III

3. (A) Write the statement of Poisson Jenson formula. **2**

(B) Define Exponents of Convergence. **2**

(C) Let G be R bounded Dirichlet Region then for each A$\in$ G there is a Green's Function on G with singularity at a.

(word limit 200-250) **4**

OR

Find the order of the function cos Z.

(D) State and prove Jensen's Inequality.

(word limit 400-450) **12**

OR

State and prove Hadamord's Three circles theorem.

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UNIT-IV

4. (A) Write the statement of Bieberbouh's conjecture. **2**

(B) Write the statement of the Great Picard theorem. **2**

(C) State and prove Little Picard Theorem.

(word limit 200-250) **4**

OR

Let f be an analytic function in a region containing the closure of the disc D = {Z : | Z | < 1} and $f(0) = 0$ $f'(0) = 1$. Then f(D) containg a disc of radius.

(D) State and prove Montel caratheodary theorem. *(word limit 400-450)* **12**

OR

State and prove Schotky's theorem.

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